We Really Need to Learn More about the Physical World

Roger PENROSE and SATO Humitaka

Physics and Architecture

SATO Humitaka: Yesterday evening in your speech for my birthday you said something rather interesting regarding Japanese culture and certain unique Japanese skills.... Could you expand on that?

Roger PENROSE: My wife and I were looking around *Ginkakuji* temple, and I was struck by the way the wood is fitted together so precisely. And not just the precision, but the delicate, beautiful artistry of it as well as the precision. Very Japanese, I thought. It had a garden around it, up on a hill, landscaped with trees and moss, and the sand is made to look like waves....

SH: Like the sea?

RP: Yes, like the sea, but in straight lines. And there's a mound—a cone, but with a flat top. And then we looked around inside the temple.... I mean, when you see architecture, it's so delicate and precise. Even things like railway trains, the way they work and fit together so well. In any other country, there's always a gap; the platform is at a different height from the train, and so on.

SH: They don't seem to care, do they? (laughs)

RP: In Japan it's always exactly right. (laughs)

SH: Yes, even when it comes to designing televisions. Sony's designs, for example, are so frightfully correct. And the reliability of components, too—the Japanese pay such close attention to detail. The gardens in Kyoto are so compact compared with those of English mansion houses, where the gardens are so big....

RP: The gardens that one might see in England, you mean?

SH: Yes. Here it's all so compact, but we still want to put everything in—even the sea and mountains and rocks.

RP: Even the scenery, that's true.

SH: It's quite a different thing. In Japan, we traditionally express not only the parts, but the whole of things.

RP: Yes, yes.

SH: When I was in England last October, I became interested in Sir Christopher WREN, NEWTON's colleague. I learned he was the architect of many big buildings....

RP: That's right, he designed St. Paul's Cathedral, of course, and a number of buildings in Oxford. He was in Oxford, at our Wadham College. He was a scientist as well as an architect.

SH: Architecture, as a design process, must be very precise in part, but the whole structure is also very important. It is more than the sum of its parts. Which makes it a very nice job! (laughs) We physicists have only to concentrate on individual parts. (laughs) But you now are becoming something of an architect of physics....

RP: Well, physics, of course, as a whole, has to have a unity. Still, there is something about Japanese architecture—in whatever country one encounters it. I remember in Princeton, there is a building, which I believe was designed by a famous Japanese architect....

SH: Is that to say you see an analogy between physics and architecture?

RP: Well, it's certainly true that physics requires a feeling for artistic values. And not just physics—in any other science, one has to be sensitive to the beauty of things. This seems to be quite important. Particularly in mathematics, one doesn't know what one's doing otherwise. In pure mathematics, that's the driving force in a way: one does it because of the pleasure it gives, for the aesthetic qualities of the subject. But physics is not quite the same thing. One's trying to find out how the world works, and it's not so obvious that artistic values are going to be important. But, nevertheless, it seems to be true, that the important and deep theories are also works of art. EINSTEIN's General Theory of Relativity has an incredibly beautiful structure. And I would say that quantum mechanics in many aspects has this extraordinary elegance about it. The question is, of course, why should that be? But to be sensitive to these things is valuable in doing science, in doing physics. It has a great value.

Human Understanding of the "Abstract"

SH: Is the beauty of pure mathematics something a non-specialist can feel? Or do we need to be specially trained?

RP: That's a good question. Mathematics is a very esoteric thing. One really has to be an expert to appreciate some aspects, and even one mathematician may have trouble appreciating what another mathematician is doing in a different area. It's the sort of thing that's very hard to communicate. And certainly very difficult for people who are not mathematicians—the general public—to appreciate for its artistic values. Nevertheless, it is possible to appreciate that such things exist, to see through simple examples that they have this aesthetic quality. One doesn't need a great deal of mathematical understanding to see that. And yet there is always a great frustration in mathematics that one may find something of tremendous beauty and then try to express this to somebody else who can't really appreciate it. That can be very disappointing in a way, because it's difficult for other people to appreciate fully what these things might mean.

SH: Pure mathematics may be too specialized, but geometric symbols or geometrical beauty and simplicity remain accessible to non-specialists.

RP: True. I have always found it remarkably ironic in a way. When people ask me to explain ideas to the general public, they say, "Well, use a lot of pictures. Use geometry to get the ideas through." Yet students in mathematics sometimes have great difficulty with geometric arguments; they are much happier with the calculations. Very often, appreciating geometry is much harder for professional mathematicians. Of course, some mathematicians can do the geometry, but they're the exception. I certainly found this when I was an undergraduate: very few of us found the geometry easy; most found the geometry much more difficult.

SH: Some inspiration is needed, even to solve an exercise....

RP: It's not so automatic. One really does need to think, I suppose.

SH: With algebra, there are basic techniques, how to start, et cetera... which maybe also distances it from the general public.

RP: Yes, that's true. But people can be very different, too. In mathematics, I've found that some people will react very differently from other people. I tend to think more geometrically. But I find I'm in the minority. There are very few people who find geometry an easy way to think.

SH: Then how is it that ordinary people have no trouble in visualizing a circle, for example, which is really such a very abstract concept?

RP: You mean a real circle, not just an approximate circle that one would draw? To appreciate the notion of an abstract circle, that's a Platonic question. People have this ability to appreciate the abstract thing... that's a good question.

SH: Circle or triangle or straight line—nobody can say they don't exist, but the concepts themselves are so abstract.

RP: Very abstract. So why do ordinary people not find that difficult? Is it the appreciation that it is only an approximation? When one thinks about a circle, the ordinary person may not realize the difficulty that even the boundary of that table is not really a circle.

SH: No, it's not. But in their minds, it's very exact.

RP: But obviously one could think about these things without really knowing what the problems are. It's only a mathematician who'd start worrying what a circle really is. Or a philosopher, perhaps.

SH: Still it seems to me that many people have very abstract concepts.

RP: I sometimes get letters from people on all sorts of strange subjects. Quite recently someone wrote to me claiming that *pi*—π—was not a constant. He thought that *pi* could be a variable, so that it might be a function of time. So I had to write to him and say, look, *pi* is a mathematical number. It couldn't be anything else; there are all sorts of mathematical formulae for *pi*, which do not directly refer to the length of the circumference of a circle. This relates to your question about an abstract circle. You see, he thought that each different circle might have a different *pi*. I had to try to explain to him that even in non-Euclidean geometry, *pi* itself is one thing. It's not that *pi* is something other than the value we give it, but whether the ratio of the circumference to the diameter is actually *pi* or not.

Not the Tools, Not the Materials

SH: Do you remember, at the tea ceremony at Urasenke the day before yesterday, the name of the tea house written 600 years ago by the tea ceremony founder, "*Shasen*"—*sha*, meaning to throw away, and *sen*, a net to catch fish. This comes from the Chinese classic *ZHUANGZI*: that people must catch fish to survive, but after catching the fish, they should throw away the net.

RP: Throw away the net?

SH: Yes, the implication is more abstract: that language is like a net. We need language for communication, but the important thing is communication. So throwing away the net also implies dispensing with language. It may be necessary, but we shouldn't depend too much on it.

RP: That's like our saying "kick away the ladder." One climbs up somewhere and then one doesn't need the ladder. It's a means to achieve an end, but once achieved, one doesn't need the means anymore.

SH: No need to store so many tools!

RP: I see, yes!

SH: Keep things simple.

RP: And minimalist.

SH: And even language should be minimized!

RP: Well, I can understand that. It's the ideas, not the way that one expresses them. Of course, in mathematics this is very true. I mean people sometimes call mathematics a language, which I myself don't see. Okay, you use certain symbols and certain operations, which you write in a particular way, but that's not the important thing. Exactly how you write it is really totally unimportant. It's the concept that underlies what you write. So, in a sense, one is abstracting what's beneath, which seems quite similar.

SH: Yes, very similar. Tea ceremony also aims at a way of life in which is simpler is better, not accumulating too many tools.

RP: The word "crutch" is sometimes used—it's a way to prop oneself up. But then afterwards one doesn't need it. I can appreciate that.

SH: Algebra is also like something to catch fish: we need it to find something, but when that thing is found, we can throw the tool away. Tea ceremony culture manifests such simplicity. Their gardens and tea houses are utterly simple and beautiful....

RP: Yes, yes, reduce to as little as is necessary. I worry about this, because I go around with all sorts of rubbish in my house—far too many belongings. My wife tells me I should throw things away, but I have great difficulty . . . .

SH: Women better appreciate simplicity. (laughs)

RP: Yes, I think it's true. But on the other hand, she has more clothes than I have, so it's almost the same.

SH: We see this in the history of Buddhism. Buddhism was a cultural import from China; it's not indigenous to Japan. So in the first era, we imitated Chinese buildings. All the pillars and walls were painted bright red. Even now, if you go to China, you can see the original style, always painted red and it looks brilliant. But soon the Japanese people began to prefer something simpler, and now we see the traditional buildings are all plain wood.

RP: Yes, but they were going to paint Ginkakuji, the "Silver Pavilion" silver. Did they never do it?

SH: They papered it with silver at first, but it peeled off. Then maybe that generation couldn't afford to replace it. And incidentally they began saying that this was much more beautiful, the simple wooden wall.

RP: What's the relation between Shintoism and Buddhism?

SH: Shinto is a more traditional religion and has no tradition of painting. All Shinto shrines are just of plain wood. It's interesting that in Shinto shrines are always supposed to be new. Every 50 years or 100 years, they rebuild them.

RP: They tear them down?

SH: And reconstruct them in exactly the same shape. So it's the shape that's important, while the wood should be always new. That's how they've lasted for a thousand years.

RP: So the concept is preserved, but not the actual material?

SH: The material is not important.

RP: That's very consistent with quantum mechanics, isn't it? (laughs) This electron and that electron are identical, so it doesn't matter which one. Whereas the structure, of course, is the important thing.

SH: No identity of substance....

RP: No identity, that's right. So that seems very modern, doesn't it?

SH: Yes, like group theory. So the Shinto shrines in Ise or Izumo, they always look new. What they do is they continually plant and maintain trees for the next shrine around the existing one. Then they cut down these trees and rebuild the next shrine.

RP: I see. So they use those trees?

SH: Yes, they self-supply the trees.

RP: But temples, they keep the same buildings. So Buddhist temples are sometimes very old as material, but in the case of shrines, it is always new.

SH: In Japan there is no tradition to use rock for building shrines or palaces—only wood. So there are no ruins from ancient times. In the past they did not so much try to preserve the substance or material itself, but would replace it with an identical structure.

RP: Is this to do with wood deteriorating, or is it just an idea?

SH: Certainly there's also the technical aspect, that it's difficult to maintain a wooden building for more than a thousand years. Maybe they thought it better to replace them more frequently.

Molecular Physics and Consciousness

SH: Now, shall we talk more about your work, such as "tiling" or your interest in the mind?

RP: Well, people seem surprised that I became interested in questions about the mind when I have been doing physics and mathematics. But in a sense, questions of the mind have interested me for a long time anyway. The viewpoint expressed in my books is one I've held for many years, since I was a research student and first learned about GÖDEL's theorem and TURING machines.

When I was in my first year as a graduate student in Cambridge, I went to lectures on GÖDEL's theorem, which was mathematical logic, and on TURING machines. These were not what I was supposed to be doing, as my research was in pure mathematics. On the one hand, GÖDEL's theorem seemed to me to make it clear that the way we understand mathematics is not just by means of given axioms—because they are never going to be sufficient—one always has ways of transcending the system of axioms. Even if one believes the axioms, one can also believe something that is not the consequence of axioms. This question of axioms is basically a question of computations: any logical system or mathematically formal system is something one could put on a computer, a TURING machine.

This view that some mathematical understanding lies outside what can be achieved by computation I must have formulated when I was in my first year as a research student. But it was just a point of view. I didn't think much about it—we all have our different ways of thinking about philosophical questions. Then I also learned about quantum mechanics from DIRAC, which was a great experience—they were a wonderful set of lectures—and about relativity from BONDI—also brilliant lectures, though in quite a different way. These were not my subjects at all at the time, but they had a great influence on what I did later.

The reason for my actually writing up my views about the mind not being a computational process was basically that I saw a television program where various people were taking a rather extreme computational position. Marvin MINSKY in particular. What they were saying was perfectly logical if one believed that all we do is computation. But I had my reasons for not believing that and since I had an earlier idea to write something at a popular or semi-popular level about science expressing my own very different view, this gave the proposed work a particular focus. So I decided to present my viewpoint in *The Emperor's New Mind* [Oxford University Press, 1989], which I hadn't seen anywhere else, but which seemed important. And one thing led to another. I hadn't anticipated the reactions I got from certain quarters—of course, I wasn't expecting people to get so angry. So the second book *Shadows of the Mind* [Oxford University Press, 1994] was meant to address what I thought were misunderstandings on their part. But then people still misunderstood things, because people still want to get angry. It really takes up too much time and it's not what I want to do. I'm much more interested in physics.

I would take the view that we're not going to make much progress on understanding what mentality is, what the mind is, until we know much more about what *physicality* is. Our picture of the physical world still has a long way to go. A lot of very important things are completely outside our present understanding. Particularly, the question of quantum mechanical state reduction, or if you like, how small-scale of quantum phenomena relate to large-scale phenomena.

SH: Usually, people in quantum mechanics do not consider the mind—this seems a peculiar combination. If I understand you correctly, you're saying that the mind suddenly appeared? This seems to be a bit strange!

RP: Suddenly? I don't think that it was sudden, just like that. It's making use of things that are out there in nature, and must have taken a long time to evolve. And I don't think of consciousness as an on-or-off thing: it's something that evolved gradually and it has a selective advantage. So creatures possessing this quality of understanding, which requires some awareness, had an advantage over creatures that did not. But I don't think this was a sudden process. I think it was something that took a long long time, much more primitive aspects of which must be present in other creatures quite far down in the animal kingdom. I don't think of it as a specifically human quality.

SH: All right, not a uniquely human quality, but anyway, do you think that the mechanics of consciousness is created through interaction with the outside?

RP: Well, that certainly is important, but I don't think it's the crucial thing. Some people regard interaction with external realities as being an essential part of consciousness, but I don't see that necessarily. Certainly in mathematics, one does an awful lot of internal thinking, which has little relation to the outside world. There's a lot to get on with completely internally. Of course, to get started, one picks up some stimuli from the outside world, and all the time one uses analogies to the outside world, but interaction with the outside is not essential.

SH: Sort of internal observation? Some processes are internal, but consciousness itself is not outside the system.

RP: One can't have a clear dividing-line, but I don't think of it as outside. It's funny. It's something in the outside world—I mean, it is out there, potentially out there, in the sense that our brains are organized to take advantage of certain potentials in the outside world. No, I don't think of consciousness as something entirely internal. It's potentially in nature, it's potentially in the way the world operates, and beings that somehow can take advantage of this potential have advantage over those who don't.

Focus on Gravity

SH: Another point that surprised many of us is that usually quantum mechanics is not directly related to general relativity, but you think that fundamentally gravity or space-time properties are always important.

RP: At some level they're intimately connected. At least I've certainly found it useful to apply quantum mechanical ideas very basically in relativity. This has to do with clocks, because relativity is fundamentally to do with time. It's the metric that basically determines time through clocks. The length of the world-line is the time measured along the world-line. And if one wants a good clock, one turns to quantum mechanics, because it's basically the relationship between mass and frequency, which is what one has in clocks. The most accurate clocks are fundamentally quantum mechanical objects. So somehow maybe there is a connection between quantum mechanics and general relativity. Without a precise measure of time, one couldn't have a clear notion of what a space-time is.

SH: Many people think the relation between space-time and quantum mechanics only matters for extreme states, such as Black Holes or the Big Bang. But you're saying that even in ordinary processes the connection between space-time and quantum mechanics is important. That's a unique point.

RP: That's right. I don't think I remember hearing anybody else saying that. State reduction is fundamentally important. The way in which the micro-world and the macro-world relate is all through state reduction. Otherwise, you would never have any correspondence between classical entities and

atoms, molecules and so on—our quantum mechanical entities—and yet quantum entities fit together and produce classical entities. There's a paradox there. Niels BOHR more or less gave up and said, well, you have a classical world and a quantum world, and your measuring apparatus is supposedly classical, only somehow it doesn't explain how, on the one hand, nature has these quantum mechanical ingredients and, on the other hand, things which behave in a classical way.
So this bridge from one to the other is a fundamental part of the way the world operates. If state reduction actually happens in the world, the most likely place for it to happen is in connection with gravity. There are people who think about these issues and worry about quantum mechanical rules being modified at some level, yet many of them don't use gravitational schemes. But at some stage, they eventually relate their ideas to gravity.
In my talk today, I mentioned John BELL because—although he never specifically used gravitational ideas until very late in his life—he had intuited that here was the most promising place to look for something different. There were other reasons, too: gravity is the only field that directly affects space-time structure. Other things do so indirectly, via energy momentum, but gravity is different; it directly affects space-time structure. It is space-time structure, in a sense. There's something quite different about gravity, so it seems reasonable that when quantum mechanics addresses gravity, different rules may well apply.

SH: I think that most physicists—myself included (laughs)—feel somewhat disconcerted that if the ordinary atom's gravitational effect on space-time is negligible, then space-time can be considered fixed and unaltered by atoms. Absolute space-time, essentially. But you don't think so.

RP: Quantum mechanics works fine if one is prepared to consider the SCHRÖDINGER equation as describing everything one needs. But at a certain level, one doesn't use the SCHRÖDINGER equation; one takes measurements to do something different. What I'm saying is, when one moves from one description to the other, that's when gravitational effects really do need to be brought in. People think they are very small—fair enough—I can understand how they think gravity is just a force, an extremely weak force. And since it is so weak, why should it affect anything? But I'm thinking about it a different way around.

SH: Classically, the effect is small. But we still don't know about quantum action in space-time.

RP: Yes. We've been thinking about how quantum mechanics might apply to just another force, not in terms of the very basis of quantum mechanics and how that's affected by space-time structure. The very way one uses quantum mechanics depends upon knowing what time displacement is, and one starts to get into a problem with that when one looks at gravitational fields that must be dealt with in superposition according to quantum mechanics.
Yes, I'm looking at things a different way around, because the way people have been thinking about the combination of quantum mechanics and gravity has been in terms of two other things: one is cosmology and the Big Bang—also the Big Crunch, maybe, if there is one—and singularities in Black Holes, where one is forced to look for some connection between quantum mechanics and general relativity.

SH: I think most people would agree to that.

RP: It's important. But as I was saying in my talk today, there's a great puzzle here. The asymmetry of time, the difference between the beginning and the end, is huge. Yet if gravity is just another physical field where one applies ordinary quantum mechanics, why do we get this gross asymmetry in time? There's nothing else which does that.

So what I'm saying is, this has to be completely different from what one has seen in other physical theories, but people think these effects are negligible because they're looking at it the wrong way around. It's the effect of gravity on quantum mechanics, not the effect of quantum mechanics on gravity.

The other place where I would expect that quantum gravity is likely to be important is much less esoteric. It's happening all the time in biology. One can't make sense of quantum mechanics without state reduction, as a physical process. One can make sense of it as just a piece of mathematics. SCHRÖDINGER equation is evolved, but then it gives us things we don't believe, like cats being alive and dead at the same time.

SH: I feel that your image of space-time is very rich. Sometimes, when people talk about space-time, space means nothing. An empty structure, nothing there. But it seems your image of space is more structured.

RP: There's a real thing there. But this is how general relativity works, as a very objective picture of space-time as a thing, which satisfies very clear equations.

SH: A physical entity.

RP: Not just the absence of things..

SH: Which makes a crucial difference! (laughs)

RP: But I think this is what EINSTEIN was saying. Well, he was driven into this position, I suppose. It was actually MINKOWSKI who said it first—that one has to combine space and time. But to make sense of general relativity, or even to make sense of special relativity, one needs to have some very objective picture of space-time as something out there. Otherwise, it's hard to make any sense of it.

I always thought "relativity" was a very bad word somehow. Because relativity creates the wrong impression; it implies that it somehow doesn't matter, that everything is relative. And yet there are these absolute notions. Space-time is an absolute notion; even if space and time individually are relative notions, space-time is objective. I'd imagine this view is common among relativists, but I may be wrong.

SH: But then there's ZENO's paradox, which we usually take as absolute. The paradox that the criterion of existence is to mentally picture something in space. So in order to try to think about real space-time, we have to situate this space-time in space. (laughs)

RP: From the point of view of visualizing it?

SH: People always try to see the existence of something.

RP: But with relativity, most often people just calculate and write down equations, without asking for pictures of what's happening.

SH: That requires some training. It's probably too abstract for ordinary people, who think that the existence is all that we image that something in space, that always becomes very difficult, that space-time itself is a physical entity, or space. For example, it's very difficult to imagine that space-time doesn't exist, so the creation of space-time is always a trouble. (laughs)

RP: If one works with relativity, one can easily reveal the real structures.

SH: Yes, of course, but the image is very difficult!

RP: True. It's hard to think about four dimensions in a very accurate way.

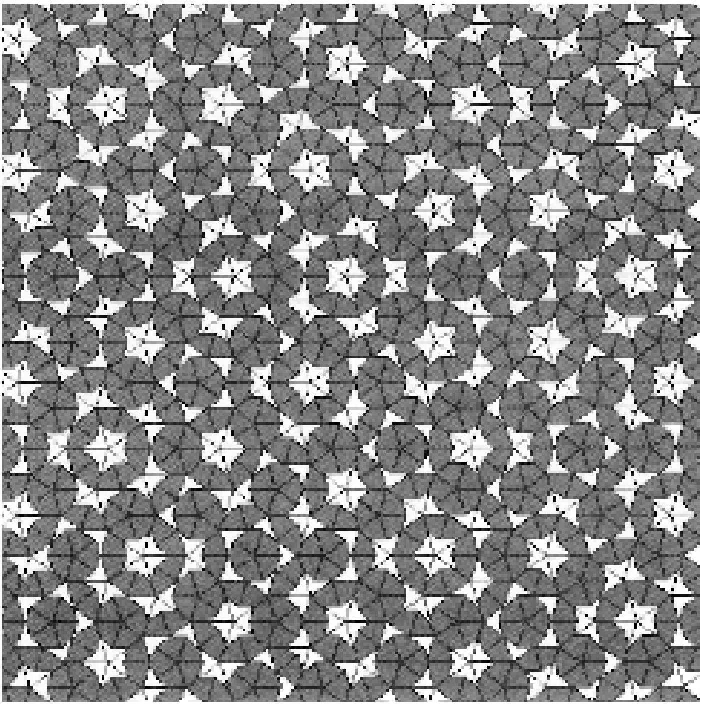
SH: Why would human consciousness have been created this way?

RP: Well, that comes back to your "sudden" questions—is it something which came about suddenly? Again, I don't think it's a specifically human quality. I don't know how far down in animals, but I would say that anyone who has owned a dog would find it hard to imagine that there isn't some kind of consciousness there. And I have very little doubt that apes, for instance, must have awareness. Elephants I'm sure do too, from things I've seen. But I'm sure it goes much more down below. I suspect there are some qualities of consciousness that go very far down in the animal kingdom. Whether fish have much consciousness, I don't know, but I don't see why not. It could be much less—there's a quantitative aspect to it—but it must be valuable to the way animals behave.
Having some awareness of what's happening out there makes them behave more effectively than if they had no such awareness. It's selectively advantageous to them, so it gets developed. But the potential must have been there all the time, otherwise they couldn't draw upon it. Like in much of natural selection, something offers an advantage at one point, which enables the development of certain structures, and then these are found useful for something else. I imagine it must be like that with consciousness itself. Maybe one-celled animals don't have any (laughs), nevertheless they have structures with potential to develop it. Just maybe, and this is a pure guess, they might take advantage of some kind of quantum coherence, which even without consciousness can be valuable to these creatures. But that's all very much guesswork.

Tilings and ESCHER

SH: Lastly, I would like you to speak about your "tiling," and perhaps also about M.C. ESCHER and aesthetic issues.

RP: Well, I should explain first of all that my father's father was a professional artist. He was a portrait painter, very representational, and came from a strict Quaker family. My father was one of four brothers, and they were all very capable, artistically; he used to do pen-and-ink drawings, and painted in oils. One of his brothers became quite a well-known artist; he was a surrealist painter and knew people like PICASSO and Max ERNST. So I suppose there is something of an artistic background in my family.
But as for the tilings, I used to doodle just for fun, designing patterns which would repeat—complicated things, different shapes that would make formal repeating patterns. One needed many of them before they would repeat. I was also interested in hierarchical structures, in which the pattern would appear at a larger scale. This was just playing around—there wasn't a feeling that this was science or anything.
One thing that must have been influential, although I didn't know it at the time, was a book my father had of KEPLER's works. Among these was a picture with many tiling patterns, some of them involving pentagons. I hadn't been thinking about them particularly, but it must have created the feeling that maybe pentagons were things that one could use for interesting designs. So I suppose that had some influence on me.
Then somebody wrote me a letter from a university in London, which has as a logo a pentagon subdivided into six other pentagons—one in the middle and five



Penrose's Tiling

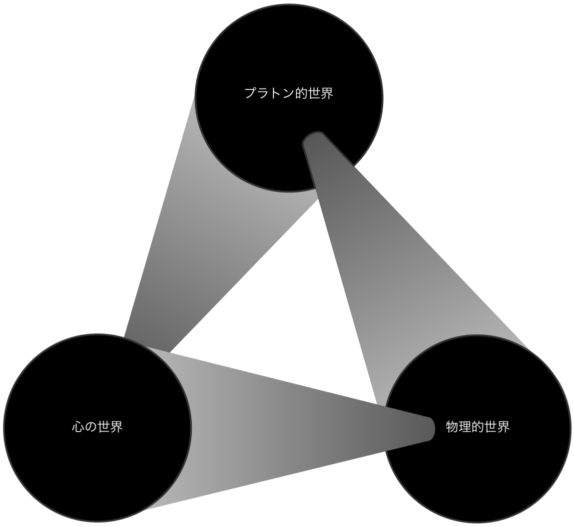
around. And I wondered what happens if one iterates that many times. One gets gaps and one has to think of ways to fill in the gaps. There's also a Japanese artist....

SH: ANNO Mitsumasa?

RP: Yes, he did something very similar—deciding how to fill the gaps. One can produce pentagons in a kind of hierarchical scheme, but there are always spaces between them. And when these spaces get big, one has to think about what to put in the spaces and make a choice whether to do it one way around or the other way. He did it the other way around, which was unfortunate, as that doesn't work so well. The other way, which I was doing, actually enables one to develop these things.

The story gets somewhat complicated, but I produced a non-repeating pattern with pentagons, and then somewhat later realized one could force these patterns into a kind of jigsaw puzzle. If one modified the shapes of the pieces a little, then one could assemble them in this way. And this led to six different shapes, which would force one into a non-repeating pattern.

Simon KOCHEN, who was visiting the Mathematical Institute in Oxford from Princeton, reminded me of Raphael ROBINSON, who had a set of six tiles that would tile a plane in a non-periodic way. He also mentioned that ROBINSON tried to keep his numbers down to a minimum. He had this non-periodic tiling based on squares—with modifications, but basically squares—and he had six different tiles to force non-periodicity. When I saw this again, I knew I could do better: my tiles were also six, but there was a



Penrose's Three-world's Triangle

redundancy; one could glue two pieces together and get it down to five. This was something of an improvement over what he had, but then I started thinking about it even more. And I realized one could get it down to two. But it's hard to give a simple explanation for why or where they came from.

The connection with ESCHER was different. Unfortunately, ESCHER never saw these tiles, because he died too soon. I'm sure he would have done wonderful things with them. When I was in my first year as a research student in Cambridge, I went to the International Congress of Mathematicians in Amsterdam and I saw a brochure of something that looked very strange to me. It was the catalog of an exhibition of ESCHER's work in a museum in Amsterdam, and I went to see it. I had never heard of ESCHER before. I became fascinated and tried to develop paradoxical designs.

Eventually I produced the "tribar"—which this triangle ["three-worlds triangle"] on the front of this book, *The Large, the Small and the Human Mind*, [Cambridge University Press, 1997] is based on—and I showed it to my father, who then started to produce all sorts other impossible buildings and things. He eventually produced this staircase, which goes around and around. We then wrote this into a paper and sent a copy to ESCHER, because he had really started us off thinking about these things. Yet the specific things that we had, he didn't—he hadn't seen before—after which he developed the staircase into «Ascending and Descending», one of his most famous works. And «Waterfall», which was based on our triangle.

On a later occasion, I actually visited ESCHER and showed him some of my tilings, which were not the non-periodic ones but other ones that were still quite complicated. Subsequently, the very last picture he ever produced, as far as I know, was based on this type of arrangement I had shown him. So that was an independent connection with ESCHER: impossible objects and those particular tiles. It's rather sad that he didn't live a bit longer to see the non-periodic ones.

[This dialogue took place in Kyoto on April 10, 1998.]

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